Elements of heat transfer

Jayadeb Dash.

Heat treanefer is a breanch of thermal science which deals with analysis of rate of heat transfer and temperature destribution taking place in a system. when temperature gradient exist.

→ The heat tranefer takes place between system to surrounding or vice versa when a temperature gradient exist bet^n them.

→ A heat treanefer analysis must be accounted for design of boilers, condensers, evaporators, heaters refrigerators and heat exchangers.

→ The heat tranfer from a higher temp region to low temp region.

→ There are three modes of heat transfer

(i) Heat conduction

(ii) Heat convection

(iii) Heat Radiation.

Heat conduction

→ The heat treanfer takes place in a solid medium from one point to another point when a temperature defference exist bet^n them is called conduction.

→ The heat conduction is the transfer of internal thermal energy by the collision of microscopic particles and movement of electrons with the body.

Fourier low of heat conduction

→ The fourier low states that the rate of heat conduction per unit area is directly proportional to temperature gradient

$$\frac{\dot{Q}}{A} \propto \frac{dT}{dx}$$

$$\Rightarrow q \propto \frac{dT}{dx}$$

$$\Rightarrow q = -K\frac{dT}{dx}$$

$$\Rightarrow \dot{Q} = -KA\frac{dT}{dx}$$

$\frac{\dot{Q}}{A}$ = heat flux = Rate of heat conduction per unit area (W/m²)

$\frac{dT}{dx}$ = Temp gradient (°c/m)

A = Area normal to heat flow.

K = constant of proportionality or Thermal conductivity (W/mK)

→ The rate of heat conduction through a medium depends on its geometry, thickness and material of the medium as well as temperature difference.

→ The -ve sign indicate the heat always flows in the direction of decreasing temperature Thus the temperature gradient $\frac{dT}{dx}$ become negative.

Thermal conductivity

→ It defind as the rate of heat transfer through a unit thickness of material per unit area per unit temperature difference.

$$K = \frac{\frac{Q}{A}}{\left(\frac{dT}{dx}\right)} = \frac{q}{\left(\frac{dT}{dx}\right)} \quad (W/m\cdot K)$$

: Heat convection:

→ The heat transfere occur between a surface and adjacent moving medium, liquid or gas when they are at different temperature.

→ The convection heat transfer comprises of two mechanism. The first is the transfer of energy due to random molecular motion (diffusion) and the second is the energy transfer by bulk motion of the fluid.

Newton low of cooling

It state that the rate of heat transfer is derectly propentional to the temperature difference betn a surface and fluid.

T_s = surface temp
T_f = fluid temp

$$\frac{\dot{Q}}{A}\,(W/m^2) \propto (T_s - T_f)\,(^\circ C)$$

$$\dot{Q}/A = h(T_s - T_f)$$ (h = constant of propertionality or heat transfer co-efficient)

$$h = \frac{\dot{Q}/A}{(T_s - T_f)}$$ (W/m²·°C or W/m²K)

→ The value of the heat transfer coefficient depends on the properties of fluid as well as fluid flow condition.

Heat Radiation

When heat transfer takes place without any medium i.e vacuum is called heat radiation.

→ In radiation the energy propagates in the form of electomagnetic waves. from high temperature region to a low-temperature region.

Stefan -Boltzmann low

→ It states that the rate of heat radiation per unit area from a black surface is directly proportional to fourth power of the absolute temperature of the surface

ie $\frac{\dot{Q}}{A} \propto T_s^4$ (T_s = Absolute temp of surface in (K))

$\frac{\dot{Q}}{A} = \sigma T_s^4$ σ = stefan Boltzmann constant $= 5.67 \times 10^{-8}\ W/m^2K^4$

$\frac{\dot{Q}}{A} = \sigma \varepsilon T_s^4$ ε = a radiative property of the surface is called emissivity

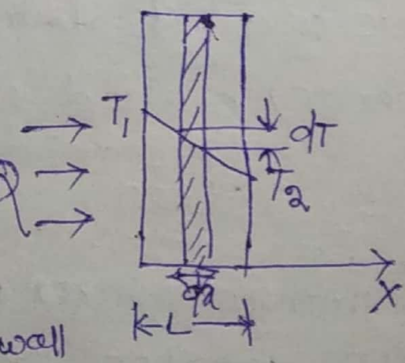
∴ The net rate of radiation heat exchange betn a real surface and its surrounding is

$\frac{\dot{Q}}{A} = \sigma \varepsilon (T_s^4 - T_\infty^4)$ (T_∞ = surrounding temp in K)

Steady-state heat conduction in solid

plane wall

consider a plane wall of thickness L and thermal conductivity K



→ Fourier equation for elemental strip of plane wall

$$\frac{\dot{Q}}{A} = -K \frac{dT}{dx}$$

$$\Rightarrow \frac{\dot{Q}}{A} dx = -K\, dT$$

$$\Rightarrow \dot{Q}/A \int_0^l dx = -K \int_{T_1}^{T_2} dT$$

$$\Rightarrow \frac{\dot{Q}}{A}[x]_0^L = -K[T]_{T_1}^{T_2}$$

$$\Rightarrow \frac{\dot{Q}}{A}[L-0] = -K[T_2 - T_1]$$

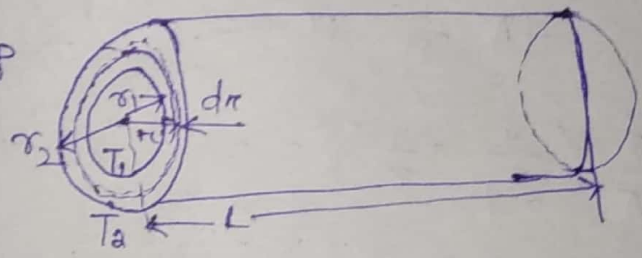
$$\Rightarrow \frac{\dot{Q}}{A}L = -K[T_2 - T_1]$$

$$\Rightarrow \dot{Q}/A = \frac{K}{L}(T_1 - T_2)$$

$$\Rightarrow \boxed{\dot{Q} = \frac{KA}{L}(T_1 - T_2)}$$

- Hollow cylender:

For an elemental strip of cylendere



$$\frac{\dot{Q}}{A} = -K\frac{dT}{dr}$$

where $A = 2\pi r L$ (area of elemental strip)

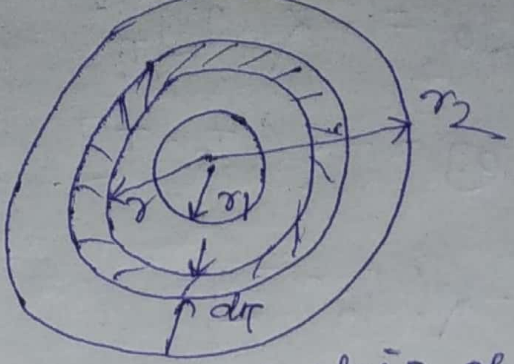
$$\frac{\dot{Q}}{2\pi r l} = -K\frac{dT}{dr}$$

$$\Rightarrow \frac{\dot{Q}}{2\pi L}\int_{r_1}^{r_2}\frac{dr}{r} = -K\int_{T_1}^{T_2} dT$$

$$\Rightarrow \frac{\dot{Q}}{2\pi L}\ln(r_2/r_1) = -K(T_2 - T_1)$$

$$\Rightarrow \dot{Q} = \frac{2\pi L K(T_1 - T_2)}{\ln(r_2/r_1)}$$

Hollow sphere



Applying fourier low of heat conduction to elemental strip of thickness dr of hollow sphere.

$$\frac{\dot{Q}}{A} = -K\frac{dT}{dr} \qquad \text{Here } A = 4\pi r^2$$

Then $$\frac{\dot{Q}}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r^2} = -K\int_{T_1}^{T_2} dT$$

$$\Rightarrow \frac{\dot{Q}}{4\pi}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = -K(T_2 - T_1)$$

$$\Rightarrow \dot{Q} = \frac{4\pi r_1 r_2 K(T_1 - T_2)}{r_2 - r_1}$$

Electrical Analogy for plane wall

$$\dot{Q} = \frac{T_1 - T_2}{(L/KA)} = \frac{\Delta T}{R_{slab}}$$

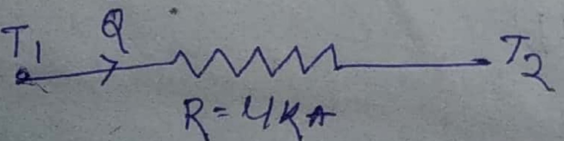
compaering this eqn with ohm's low for electrical network $I = \frac{\Delta V}{R}$

ΔT = Thermal potential difference °C or K

$\frac{L}{KA}$ = Thermal resistance °C/W or K/W

$\dot{Q}$ = Heat flow rate or heat current in W

ie



For convection heat transfer

$$\dot{Q} = hA(T_s - T_f)$$

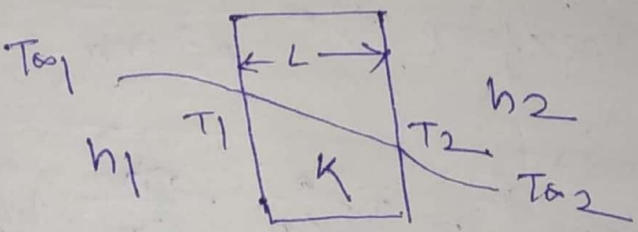
$$\dot{Q} = \frac{T_s - T_f}{\left(\frac{1}{hA}\right)} = \frac{T_s - T_f}{R_{conv.}}$$

$R_{conv} = 1/hA$ = convective resistance

$T_s - T_f$ = Thermal potential difference at the surface

For combine mode of heat transfer

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{\left(\frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A}\right)}$$



Electrical analogy for hollow cylinder

$$\dot{Q} = \frac{T_1 - T_2}{\frac{\ln(r_2/r_1)}{2\pi LK}} = \frac{\Delta T}{R_{cylinder}}$$

$$R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi LK}$$

For combine mode

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{\left(\frac{1}{2\pi r_1 L h_1}\right) + \frac{\ln(r_2/r_1)}{2\pi LK} + \frac{1}{(2\pi r_2 L) h_2}}$$

Electrical analogy for Hollow sphere

$$\dot{Q} = \frac{T_1 - T_2}{\frac{r_2 - r_1}{4\pi r_1 r_2 k}} = \frac{\Delta T}{R_{sph}}$$

$$R_{sph} = \frac{r_2 - r_1}{4\pi r_1 r_2 K}$$

for combine mode of conduction and convecti

$$\therefore \dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{4\pi r_1^2 h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{4\pi r_2^2 h_2}}$$

overall heat - Transfer co-efficient

$$\dot{Q} = \cancel{Q}\, UA(\Delta T)_{overall} = \frac{\Delta T}{\Sigma R_{th}}$$

Therefore $U = \frac{1}{A \Sigma R_{th}}$

so $U = \frac{1}{\left(\frac{1}{h_1} + \frac{l_A}{k_A} + \cancel{\frac{l_B}{k_B}} + \cancel{\frac{l_C}{k_C}} + \frac{1}{h_2}\right)}$

Log mean area

The heat tranfer through a hollow cyclinder of the same form as that of a plane wall

then $R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi L k}$

$$R_{cyl} = \frac{r_2 - r_1}{r_2 - r_1} \times \frac{\ln\left(\frac{2\pi r_2 L}{2\pi r_1 L}\right)}{2\pi L k}$$

$$= \frac{r_2 - r_1 \ln(A_2/A_1)}{(A_2 - A_1) k}$$

or $R_{cyl} = \frac{r_2 - r_1}{A_m k}$

where $A_2 = 2\pi r_2 L$
$A_1 = 2\pi r_1 L$
$A_m =$ log mean area

where $A_m = \frac{A_2 - A_1}{\ln(A_2/A_1)}$

$$\dot{Q} = \frac{A_m K (T_1 - T_2)}{r_2 - r_1}$$

Theories of radiation

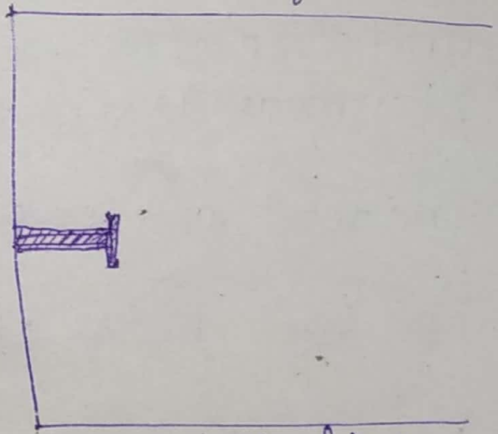
Maxwells Theory

→ According to maxwells electromagnetic theory the energy is transfered from a hot body to cold body in the form of the electromagnetic waves

→ All electromagnetic wave travel with the speed of light.

Max planck's Theory

According to max plancks concepet, the propagation of thermal radiation takes place in the form of discrete quanta called photons, each quantum having an energy of



$$E = h\nu$$

h = plancks constant.

ν = frequency of photons.

Black-Body Radiation

→ A black body absorebs all incident radiation from all direction at all wavelengths

→ At any temperature and wavelength, no body can emit energy more than a black body

→ Although the radiation emitted by a black body depend upon wavelength and temperature it is independent of direction.

→ A black body neither reflects nor transmits any amount of incident radiation.

Weinx displacement locu

It states that the product of wave length and absolute temperature corresponding to the locus of all peaks is always constant and his analytical formula is

$$(\lambda T)_{max} = 2897.6\ \mu mK$$

λ = wave length in μm

T = Temperature.

Emissive power of a black Body

→ The total or hemispherical emissive power is the amount of radiation energy emitted by a black body at a temperature per unit area per unit time over entire spectrum of wavelength It is measured in W/m2

The total emissive power of a black surface is $E_b = \int_0^{\infty} E_{b\lambda}\, d\lambda$

$E_{b\lambda}$ = spectral blackbody emissive power $W/m^2\,\mu m$

Emissivity

→It is the ratio of radiation heat flux to the emissive power of a black body

$$\boxed{\varepsilon = \frac{E}{E_b}}$$

$$\boxed{E = \varepsilon E_b = \varepsilon \sigma T^4}$$

E = Emmisive power

Irradiation (G)

→ It is the total radiation energy incident per unit area per unit time over the entire wavelength from all direction

→ or The radiation flux incident on a surface is called irradiation and is denoted by (G)

Absorptivity (α)

→ The fraction of irradiation absorbed by the surface is called the absorptivity (α)

→ It is the ratio of absorbed radiation (G_a) to incident radiation (G)

→ $$\alpha = \frac{\text{Energy absorbed by the surface, W/m2}}{\text{Irradiation, W/m2}}$$

$$\alpha = \frac{G_a}{G}$$

→ It value : $0 \leq \alpha \leq 1$

Reflectivity (ρ)

→ The fraction of radiation reflected by the surface is called reflectivity (ρ).

→ It is the ratio of reflected radiation (G_r) to incident radiation (G)

→ Its value : $0 \leq \rho \leq 1$

→ $\rho = G_r/G$

Transmissivity (τ)

→ The fraction of radiation transmitted is called transmissivity (τ).

→ It is the ratio of transmitted radiation to incident radiation (G)

→ $\tau = G_t/G$, Its value lies bet $0 \leq \tau \leq 1$

→ According to first low of thermodynamics the sum of the absorbed, reflected and transmitted radiation energy be equal to the incident radiation, that is

$$G_a + G_r + G_t = G$$

∴ Dividing each term of this relation by G

$$\frac{G_a}{G} + \frac{G_r}{G} + \frac{G_t}{G} = \frac{G}{G}$$

$$\Rightarrow \boxed{\alpha + \rho + \tau = 1}$$

White body:

→ A body is called white body when it reflects all radiation incident upon it

→ for a white body reflectivity is one ie $\rho = 1$

opaque body

→ The body which is has no capasity to transmission is called opaque body ie $\tau = 0$

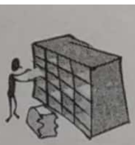
Gray surface

→ Gray surface is defind as a surface for which the monochromatic emissivity E_λ is indipendent of the wave length.

Kirchhoff's Law

→ It state that at thermal equilibrium, the ratio of the spectral emissive power to the spectral absorptivity for all bodies is constant

ie $$\frac{E_{\lambda 1}}{\alpha_{\lambda 1}} = \frac{E_{\lambda 2}}{\alpha_{\lambda 2}} = C$$



Glossary

Heat flux Heat transfer rate per unit area

Conduction Heat transfer due to existance of temperature gradient in material medium

Free convection Heat transfer due to density difference induced by temperature difference in fluids

Forced convection Heat transfer due to velocity difference induced artificially

Radiation Heat transfer due to electromagnetic waves from surfaces

Thermal conductivity Ability of material to conduct the heat

Thermal potential Temperature difference; responsuible for heat transfer

Thermal resistance Opposes the heat flow through the material medium

Heat transfer coefficient Property of ambient conditions

Black body An imaginary ideal body for radiation

Emisive power Radiation heat transfer per unit area

Emissivity Property of a radiating surface

LMTD Log mean temperature difference

NTU Number of transfer unit

Effectiveness Ratio of actual heat transfer rate to maximum possible heat transfer rate

THEORETICAL QUESTIONS

1. Enumerate the three modes by which heat can be transferred from one place to another. Which is the slowest of all ?
2. How do you define the thermal conductivity of a material ?
3. What do you understand by the terms 'convective heat transfer co-efficient' and 'overall heat transfer co-efficient'.
4. Derive an expression for heat loss in kJ/m^2-hr through a composite wall of layers (*i*) without considering convective heat transfer co-efficients and (*ii*) considering the convective heat transfer co-efficients.
5. Classify the heat exchangers according to the flow directions of fluid and give few examples of each in actual field of application.

Steam power cycle

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→ In a steam power cycle water is used as working substance.

→ In steam power cycle the phase of working substance changes alternatively.

→ The change of phase allows more energy to be stored in the working substance that canbe stored by only sensible heating.

→ The steam power plants generate a major fraction of electric power produced in the world.

Performance parameters of steam power cycle

(1) Thermal efficiency (η_{th})

→ Thermal efficiency of the steam power cycle is defind as the ratio of Net work done produced in the cycle to the heat supplied in the cycle.

$$\eta_{th} = \frac{\text{Net work don in the cycle}}{\text{Heat supplied in the cycle}}$$

$$\eta_{th} = \frac{W_{net}}{Q_{in}}$$

(2) Back work ratio (r_{bwr}

→ It is the ratio of pump work input to the work developed by the turbine.

$$\rightarrow r_{bwr} = \frac{\text{pump work}}{\text{Turbine work}} = \frac{W_P}{W_T}$$

(3) Work ratio (r_w)

→ It is defind as the ratio of net work outpu of the cycle to the work developed by turbin

→ $r_w = \frac{\text{Net work output of the cycle}}{\text{Turbine work}}$

→ $r_w = \frac{W_{net}}{W_T} = \frac{W_T - W_P}{W_T}$

→ $r_w = 1 - \frac{W_P}{W_T} = 1 - r_{bwr}$

(4) Steam rate (ssc)

→ It is also called specific steam consumptio

→ It is defind as the amount of steam required to produce 1KWh or 3600KJ of power. $SSC = \frac{\dot{m}_s\ (kg/h)}{\dot{m}_s\ (kg/s) \times W_{net}\ (KJ/kg)}$

→ $SSC = \frac{3600 (kg)}{W (KWh)}$

(5) Heat Rate

→ It is defind as the amount of heat require by a power plant to produce 1 KWh of power

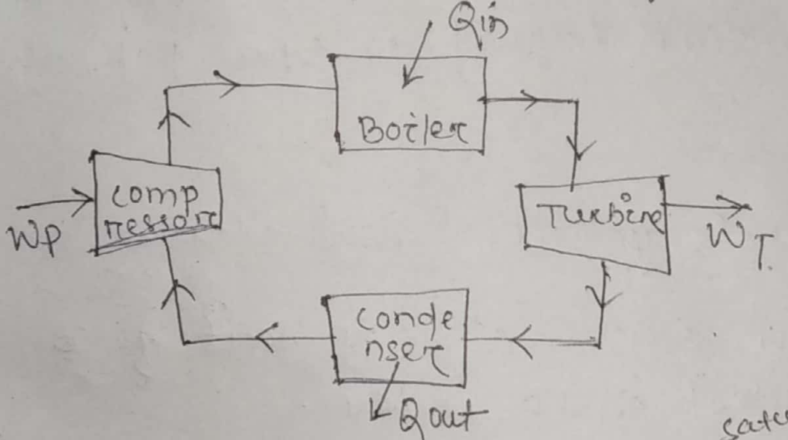
→ $\text{Heat rate} = \frac{(\text{Heat input in KJ/s}) \times (3600\ s/h)}{\text{Net power output in KW}}$

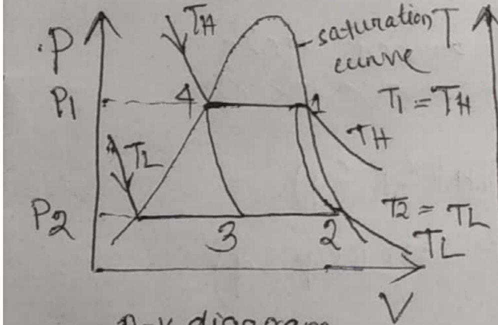
$= \frac{3600}{\eta_{th}}\ (KJ/KWh)$

Carnot steam power cycle

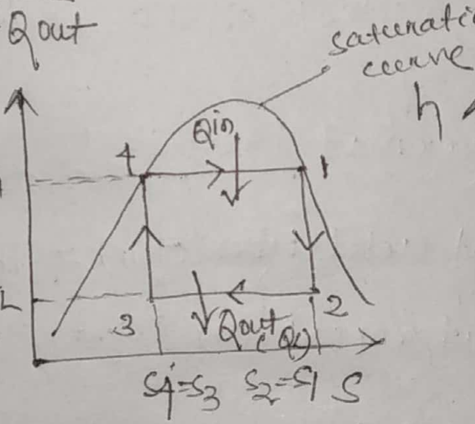
→ Carnot cycle gives the max^m efficiency.

→ It operating between two temperature limit and ~~it,~~ efficiency is indipendent of properties of working substance.

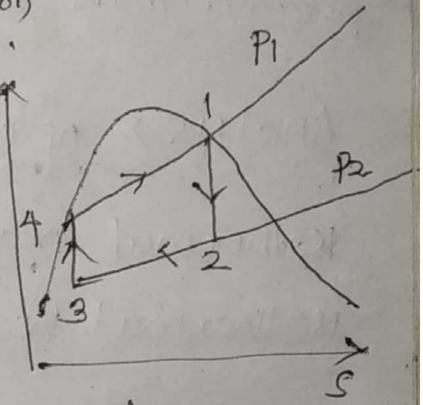




p-v diagram



T-S diagram



h-s diagram

→ Carnot cycle consist of 4-process

Operation 1-2 (Reversible adiabatic expansion in turbine)

→ Saturated steam expands in the turbine

→ The temperature lowers from T_H to T_L and pressure lowers from P_1 to P_2 and the state 2 reached in the wet region

→ During this operation W_T amount of workdone by the turbine

Analysis of rankine cycle

consider 1 kg of fluid

(i) **For boiler**

$$h_{f4} + Q_1 = h_1$$

$$Q_1 = h_1 - h_{f4}$$

(ii) **For turbine**

$$h_1 = W_T + h_2$$

$$W_T = h_1 - h_2$$

(iii) **For condenser**

$$h_2 = Q_2 + h_{f3}$$

$$Q_2 = h_2 - h_{f3}$$

(iv) **For feed pump**

$$h_{f3} + W_p = h_{f4}$$

$$W_p = h_{f4} - h_{f3}$$

Now efficiency of rankine cycle

$$\eta_{ran} = \frac{W_{net}}{Q_1} = \frac{W_T - W_p}{Q_1}$$

$$\eta_{ran} = \frac{(h_1 - h_2) - (h_{f4} - h_{f3})}{(h_1 - h_{f4})}$$

we know $T\,ds = dh - v\,dp$ As $ds = 0$

$$dh = v\,dp$$

$$h_{f4} - h_{f3} = v_3(P_1 - P_2)$$

The feed pump term ($h_{f4} - h_{f3}$) being a small quantity in comparison with turbine work (W_T) is usually neglected.

Then $$\boxed{\eta_{ran} = \frac{h_1 - h_2}{h_1 - h_{f4}}}$$

comparision bet rankine and carnat cycle

(i) Between same temperature limits Rankine cycle provide a higher specific work output than carnat cycle.

(2) Rankine cycle require a smaller steam flow rate than carnat cycle

(3) Rankine cycle has a high rate of heat transfer in boiler and condenser

(4) Rankine cycle efficiency is lower than carnat cycle.

(5) work for compression is very large compared to to the pump is carnat cycle.

Effect of operating condition on rankine cycle efficiency

→ The rankine cycle efficiency can be improved by

(i) Increasing the avg temperature at which heat is supplied

(ii) Decreasing the ~~[illegible]~~ temperature at which heat is rejected.

This can be done by

(i) Increasing boiler pressure

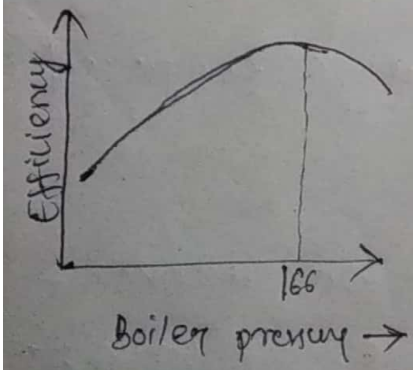
→ It has been observed that by increasing the boiler pressure the cycle efficiency tends to rise and reaches a max^m value at a boiler pressure of about 166 bar

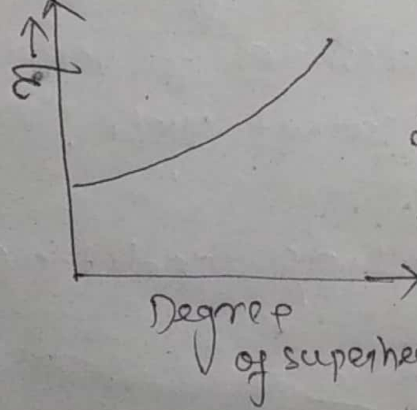
(ii) Super heating

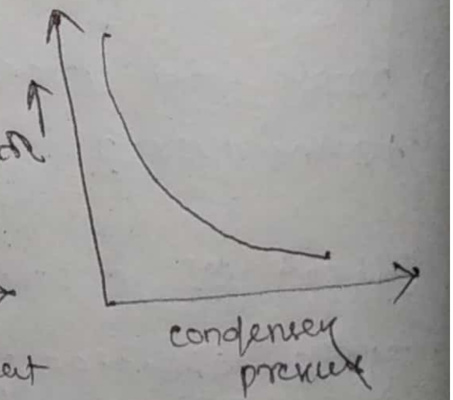
→ The steam is super heated before allowing it to expand the rankine cycle efficiency may be increased

(iii) Reducing condenser pressure

→ The thermal efficiency of the cycle can be amply improved by reducing the condenser pressure.







12.5. REHEAT CYCLE

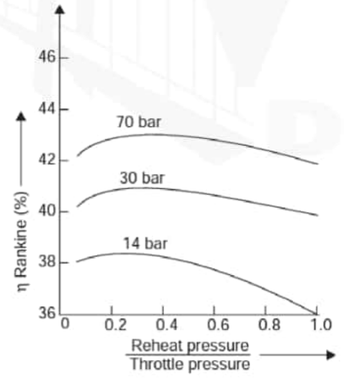
For attaining greater thermal efficiencies when the initial pressure of steam was raised beyond 42 bar it was found that resulting condition of steam after, expansion was increasingly wetter and exceeded in the safe limit of 12 per cent condensation. It, therefore, became necessary to *reheat* the steam after part of expansion was over so that the resulting condition after complete expansion fell within the region of permissible wetness.

The reheating or resuperheating of steam is now universally used when high pressure and temperature steam conditions such as 100 to 250 bar and 500°C to 600°C are employed for throttle. For plants of *still higher pressures and temperatures, a double reheating may be used.*

In actual practice reheat *improves* the cycle efficiency by about 5% for a 85/15 bar cycle. A *second reheat* will give a *much less gain* while the initial cost involved would be so high as to prohibit use of two stage reheat except in case of very high initial throttle conditions. The cost of reheat equipment consisting of boiler, piping and controls may be 5% to 10% more than that of the conventional boilers and this additional expenditure is justified only if gain in thermal efficiency is sufficient to promise a return of this investment. *Usually a plant with a base load capacity of 50000 kW and initial steam pressure of 42 bar would economically justify the extra cost of reheating.*

The improvement in thermal efficiency due to reheat is greatly dependent upon the *reheat pressure* with respect to the original pressure of steam.

Fig. 12.23 shows the reheat pressure selection on cycle efficiency.



Condenser pressure : 12.7 mm Hg
Temperature of throttle and heat : 427°C
Fig. 12.23. Effect of reheat pressure selection on cycle efficiency.

Fig. 12.24 shows a schematic diagram of a theoretical single-stage reheat cycle. The corresponding representation of ideal reheating process on *T-s* and *h-s* chart is shown in Figs. 12.25 (*a* and *b*).

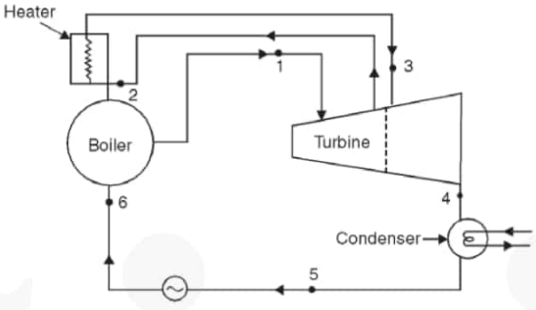


Fig. 12.24. Reheat cycle.

Refer to Fig. 12.25. 5-1 shows the formation of steam in the boiler. The steam as at state point 1 (*i.e.*, pressure p_1 and temperature T_1) enters the turbine and expands isentropically to a certain pressure p_2 and temperature T_2. From this state point 2 the whole of steam is drawn out of the turbine and is reheated in a reheater to a temperature T_3. (Although there is an *optimum pressure* at which the steam should be removed for reheating, if the highest return is to be obtained, yet, for simplicity, the whole steam is removed from the high pressure exhaust, where the pressure is about *one-fifth* of boiler pressure, and after undergoing a 10% pressure drop, in circulating through the heater, it is returned to intermediate pressure or low pressure turbine). This reheated steam is then readmitted to the turbine where it is expanded to condenser pressure isentropically.

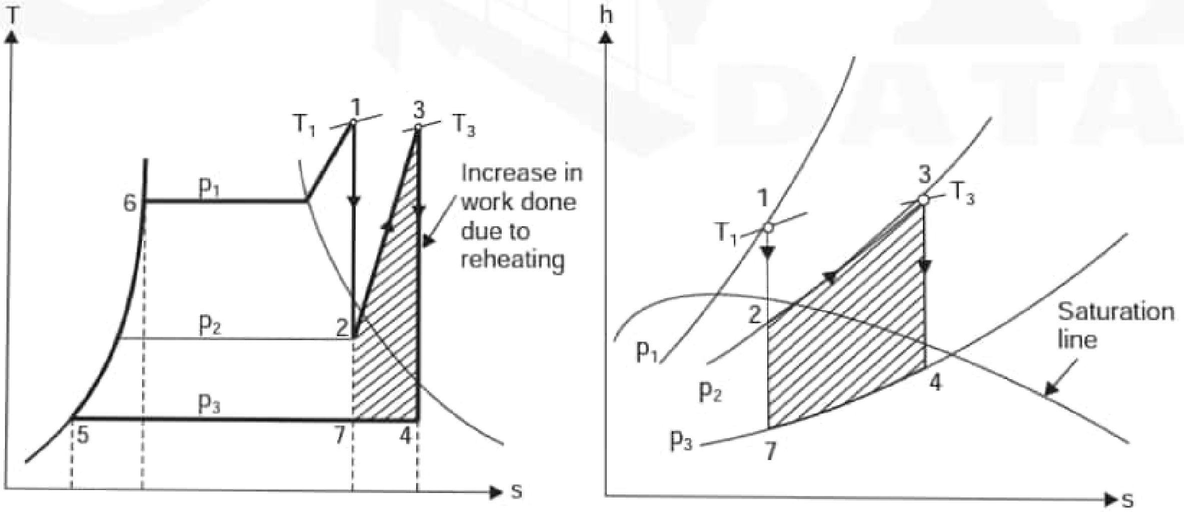


Fig. 12.25. Ideal reheating process on *T-s* and *h-s* chart.

Note. Superheating of steam. The primary object of superheating steam and supplying it to the primemovers is *to avoid too much wetness at the end of expansion*. Use of inadequate degree of superheat in *steam engines would cause greater condensation* in the engine cylinder ; while in case of *turbines* the moisture content of steam would result in *undue blade erosion*. The *maximum wetness in the final condition of steam that may be tolerated without any appreciable harm to the turbine blades is about 12 per cent*. Broadly each *1 per cent* of moisture in steam reduces the efficiency of that part of the turbine in which wet steam passes by 1 per cent to 1.5 per cent and in engines about 2 per cent.

Advantages of superheated steam :

(*i*) Superheating reduces the initial condensation losses in steam engines.

(*ii*) Use of superheated steam results in *improving the plant efficiency* by effecting a *saving in cost of fuel*. This saving may be of the order of 6% to 7% due to first 38°C of superheat and 4% to 5% for next 38°C and so on. This saving results due to the fact that the heat content and consequently the capacity to do work in superheated steam is increased and the quantity of steam required for a given output of power is reduced. Although additional heat has to be added in the boiler there is reduction in the work to be done by the feed pump, the condenser pump and other accessories due to reduction in quantity of steam used. It is estimated that the *quantity of steam may be reduced by 10% to 15% for first 38°C of superheat and somewhat less for the next 38°C of superheat in the case of condensing turbines.*

(*iii*) When a superheater is used in a boiler it helps in *reducing the stack temperatures* by extracting heat from the flue gases before these are passed out of chimney.

Thermal efficiency with '*Reheating*' (neglecting pump work) :

Heat supplied $= (h_1 - h_{f_4}) + (h_3 - h_2)$

Heat rejected $= h_4 - h_{f_4}$

Work done by the turbine = Heat supplied – heat rejected

$$= (h_1 - h_{f_4}) + (h_3 - h_2) - (h_4 - h_{f_4})$$

$$= (h_1 - h_2) + (h_3 - h_4)$$

Thus, theoretical thermal efficiency of reheat cycle is

$$\eta_{\text{thermal}} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_{f_4}) + (h_3 - h_2)} \qquad \text{...(12.11)}$$

If pump work, $W_p = \frac{v_f\,(p_1 - p_b)}{1000}$ kJ/kg is considered, the thermal efficiency is given by :

$$\eta_{\text{thermal}} = \frac{[(h_1 - h_4) + (h_3 - h_4)] - W_p}{[(h_1 - h_{f_4}) + (h_3 - h_2)] - W_p} \qquad \text{...(12.12)}$$

W_p is usually small and *neglected*.

Thermal efficiency *without reheating* is

$$\eta_{\text{thermal}} = \frac{h_1 - h_7}{h_1 - h_{f_4}} \quad (\because \quad h_{f_4} = h_{f_7}) \qquad \text{...(12.13)}$$

Note 1. The reheater may be incorporated in the walls of the main boiler ; it may be a separately fired superheater or it may be heated by a coil carrying high-pressure superheated steam, this system being analogous to a steam jacket.

2. Reheating should be done at '*optimum pressure*' because if the steam is reheated early in its expansion then the additional *quantity of heat supplied will be small* and thus thermal efficiency gain will be small ; and if the reheating is done at a fairly low pressure, then, although a large amount of additional heat is supplied, the steam will have a high degree of superheat (as is clear from Mollier diagram), thus a large proportion of the heat supplied in the reheating process *will be thrown to waste in the condenser*.

Advantages of 'Reheating' :

1. There is an increased output of the turbine.
2. Erosion and corrosion problems in the steam turbine are eliminated/avoided.
3. There is an improvement in the thermal efficiency of the turbines.
4. Final dryness fraction of steam is improved.
5. There is an increase in the nozzle and blade efficiencies.

Disadvantages :

1. Reheating requires more maintenance.

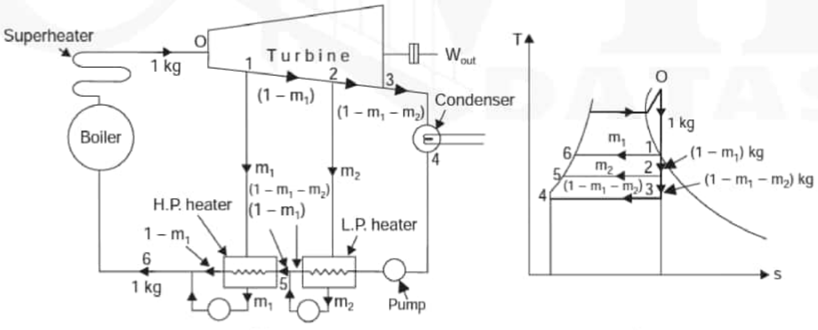
2. The increase in thermal efficiency is not appreciable in comparison to the expenditure incurred in reheating.

12.4. REGENERATIVE CYCLE

In the Rankine cycle it is observed that the condensate which is fairly at low temperature has an irreversible mixing with hot boiler water and this results in decrease of cycle efficiency. Methods are, therefore, adopted to heat the feed water from the hot well of condenser irreversibly by interchange of heat within the system and thus improving the cycle efficiency. This heating method is called regenerative feed heat and the cycle is called *regenerative cycle*.

The principle of regeneration can be practically utilised by extracting steam from the turbine at several locations and supplying it to the regenerative heaters. The resulting cycle is known as *regenerative or bleeding cycle*. The heating arrangement comprises of : (*i*) For medium capacity turbines—not more than 3 heaters ; (*ii*) For high pressure high capacity turbines—not more than 5 to 7 heaters ; and (*iii*) For turbines of super critical parameters 8 to 9 heaters. The most advantageous condensate heating temperature is selected depending on the turbine throttle conditions and this determines the number of heaters to be used. The final condensate heating temperature is kept 50 to 60°C below the boiler saturated steam temperature so as to prevent evaporation of water in the feed mains following a drop in the boiler drum pressure. The conditions of steam bled for each heater are so selected that the temperature of saturated steam will be 4 to 10°C higher than the final condensate temperature.

Fig. 12.15 (*a*) shows a diagrammatic layout of a condensing steam power plant in which a surface condenser is used to condense all the steam that is not extracted for feed water heating. The turbine is double extracting and the boiler is equipped with a superheater. The cycle diagram (*T-s*) would appear as shown in Fig. 12.15 (*b*). This arrangement constitutes a *regenerative cycle*.



(a) (b)

Fig. 12.15. Regenerative cycle.

Let, m_1 = kg of high pressure (H.P.) steam per kg of steam flow,

m_2 = kg of low pressure (L.P.) steam extracted per kg of steam flow, and

$(1 - m_2 - m_2)$ = kg of steam entering condenser per kg of steam flow.

Energy/Heat balance equation for H.P. heater :

$$m_1 (h_1 - h_{f_6}) = (1 - m_1)(h_{f_6} - h_{f_5})$$

or $$m_1[(h_1 - h_{f_6}) + (h_{f_6} - h_{f_5})] = (h_{f_6} - h_{f_5})$$

or $$m_1 = \frac{h_{f_6} - h_{f_5}}{h_1 - h_{f_5}} \quad \text{...(12.8)}$$

Energy/Heat balance equation for L.P. heater :

$$m_2 (h_2 - h_{f_5}) = (1 - m_1 - m_2)(h_{f_5} - h_{f_3})$$

or $$m_2[(h_2 - h_{f_5}) + (h_{f_5} - h_{f_3})] = (1 - m_1)(h_{f_5} - h_{f_3})$$

or $$m_2 = \frac{(1 - m_1)(h_{f_5} - h_{f_3})}{(h_2 - h_{f_3})} \quad \text{...(12.9)}$$

All enthalpies may be determined ; therefore m_1 and m_2 may be found. The maximum temperature to which the water can be heated is dictated by that of bled steam. The condensate from the bled steam is added to feed water.

Neglecting pump work :

The heat supplied externally in the cycle

$$= (h_0 - h_{f_6})$$

Isentropic work done $= m_1 (h_0 - h_1) + m_2 (h_0 - h_2) + (1 - m_1 - m_2)(h_0 - h_3)$

The thermal efficiency of regenerative cycle is

$$\eta_{\text{thermal}} = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{m_1 (h_0 - h_1) + m_2 (h_0 - h_2) + (1 - m_1 - m_2)(h_0 - h_3)}{(h_0 - h_{f_6})} \quad \text{...(12.10)}$$

[The work done by the turbine may also be calculated by summing up the products of the steam flow and the corresponding heat drop in the turbine stages.

i.e., Work done $= (h_0 - h_1) + (1 - m_1)(h_1 - h_2) + (1 - m_1 - m_2)(h_2 - h_3)$]

Advantages of Regenerative cycle over Simple Rankine cycle :

1. The heating process in the boiler tends to become reversible.

All enthalpies may be determined ; therefore m_1 and m_2 may be found. The maximum temperature to which the water can be heated is dictated by that of bled steam. The condensate from the bled steam is added to feed water.

Neglecting pump work :

The heat supplied externally in the cycle

$$= (h_0 - h_{f_6})$$

Isentropic work done $= m_1 (h_0 - h_1) + m_2 (h_0 - h_2) + (1 - m_1 - m_2) (h_0 - h_3)$

The thermal efficiency of regenerative cycle is

$$\eta_{\text{thermal}} = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{m_1 (h_0 - h_1) + m_2 (h_0 - h_2) + (1 - m_1 - m_2) (h_0 - h_3)}{(h_0 - h_{f_6})} \quad \text{...(12.10)}$$

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i.e., Work done $= (h_0 - h_1) + (1 - m_1) (h_1 - h_2) + (1 - m_1 - m_2) (h_2 - h_3)$]

Advantages of Regenerative cycle over Simple Rankine cycle :

1. The heating process in the boiler tends to become reversible.
2. The thermal stresses set up in the boiler are minimised. This is due to the fact that temperature ranges in the boiler are reduced.
3. The thermal efficiency is improved because the average temperature of heat addition to the cycle is increased.
4. Heat rate is reduced.
5. The blade height is less due to the reduced amount of steam passed through the low pressure stages.
6. Due to many extractions there is an improvement in the turbine drainage and it reduces erosion due to moisture.
7. A small size condenser is required.

Disadvantages :

1. The plant becomes more complicated.
2. Because of addition of heaters greater maintenance is required.
3. For given power a large capacity boiler is required.
4. The heaters are costly and the gain in thermal efficiency is not much in comparison to the heavier costs.

Note. In the absence of precise information (regarding actual temperature of the feed water entering and leaving the heaters and of the condensate temperatures) the following assumption should always be made while doing calculations :

1. Each heater is ideal and bled steam just condenses.
2. The feed water is heated to saturation temperature at the pressure of bled steam.
3. Unless otherwise stated the work done by the pumps in the system is considered negligible.
4. There is equal temperature rise in all the heaters (usually 10°C to 15°C).

Example 12.12. *A steam turbine is fed with steam having an enthalpy of 3100 kJ/kg. It*

Example 12.2. *In a steam power cycle, the steam supply is at 15 bar and dry and saturated. The condenser pressure is 0.4 bar. Calculate the Carnot and Rankine efficiencies of the cycle. Neglect pump work.*

Solution. Steam supply pressure, $p_1 = 15$ bar, $x_1 = 1$

Condenser pressure, $p_2 = 0.4$ bar

Carnot and Rankine efficiencies :

From steam tables :

At 15 bar : $t_s = 198.3°\text{C}$, $h_g = 2789.9$ kJ/kg, $s_g = 6.4406$ kJ/kg K

At 0.4 bar : $t_s = 75.9°\text{C}$, $h_f = 317.7$ kJ/kg, $h_{fg} = 2319.2$ kJ/kg,

$$s_f = 1.0261 \text{ kJ/kg K},\ s_{fg} = 6.6448 \text{ kJ/kg K}$$

$$T_1 = 198.3 + 273 = 471.3 \text{ K}$$

$$T_2 = 75.9 + 273 = 348.9 \text{ K}$$

$$\therefore \quad \eta_{\text{carnot}} = \frac{T_1 - T_2}{T_1} = \frac{471.3 - 348.9}{471.3}$$

$$= \mathbf{0.259 \text{ or } 25.9\%.\ (Ans.)}$$

$$\eta_{\text{Rankine}} = \frac{\text{Adiabatic or isentropic heat drop}}{\text{Heat supplied}} = \frac{h_1 - h_2}{h_1 - h_{f_2}}$$

where $h_2 = h_{f_2} + x_2\, h_{fg_2} = 317.7 + x_2 \times 2319.2$ ...(*i*)

Value of x_2 :

As the steam expands isentropically,

$$\therefore \quad s_1 = s_2$$

$$6.4406 = s_{f_2} + x_2\, s_{fg_2} = 1.0261 + x_2 \times 6.6448$$

$$\therefore \quad x_2 = \frac{6.4406 - 1.0261}{6.6448} = 0.815$$

$$\therefore \quad h_2 = 317.7 + 0.815 \times 2319.2 = 2207.8 \text{ kJ/kg} \qquad \text{[From eqn. (i)]}$$

Hence, $\eta_{\text{Rankine}} = \dfrac{2789.9 - 2207.8}{2789.9 - 317.7} = \mathbf{0.2354 \text{ or } 23.54\%.}$ **(Ans.)**

Example 12.3. *In a steam turbine steam at 20 bar, 360°C is expanded to 0.08 bar. It then enters a condenser, where it is condensed to saturated liquid water. The pump feeds back the water into the boiler. Assume ideal processes, find per kg of steam the net work and the cycle efficiency.*

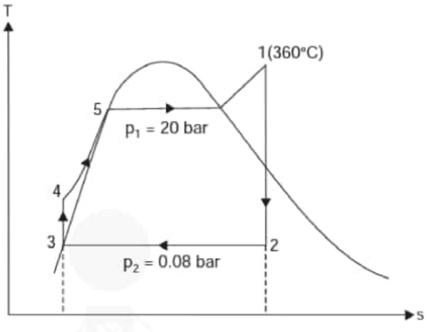

Fig. 12.7

Solution. Boiler pressure, $p_1 = 20$ bar (360°C)

Condenser pressure, $p_2 = 0.08$ bar

From steam tables :

At 20 bar (p_1), 360°C : $h_1 = 3159.3$ kJ/kg

$s_1 = 6.9917$ kJ/kg-K

At 0.08 bar (p_2) : $h_3 = h_{f(p_2)} = 173.88$ kJ/kg,

$s_3 = s_{f(p_2)} = 0.5926$ kJ/kg-K

$h_{fg(p_2)} = 2403.1$ kJ/kg, $s_{g(p_2)} = 8.2287$ kJ/kg-K

$v_{f(p_2)} = 0.001008$ m³/kg $\therefore$ $s_{fg(p_2)} = 7.6361$ kJ/kg-K

Now $s_1 = s_2$

$$6.9917 = s_{f(p_2)} + x_2\, s_{fg(p_2)} = 0.5926 + x_2 \times 7.6361$$

$$\therefore \quad x_2 = \frac{0.69917 - 0.5926}{7.6361} = 0.838$$

$$\therefore \quad h_2 = h_{f(p_2)} + x_2\, h_{fg(p_2)}$$

$$= 173.88 + 0.838 \times 2403.1 = 2187.68 \text{ kJ/kg.}$$

Net work. W :

Net work, W_{net} :

$$W_{net} = W_{turbine} - W_{pump}$$

$$W_{pump} = h_{f_4} - h_{f(p_2)} \; (= h_{f_3}) = v_{f(p_2)} \; (p_1 - p_2)$$

$$= 0.00108 \text{ (m}^3\text{/kg)} \times (20 - 0.08) \times 100 \text{ kN/m}^2$$

$$= 2.008 \text{ kJ/kg}$$

$$[\text{and} \quad h_{f_4} = 2.008 + h_{f(p_2)} = 2.008 + 173.88 = 175.89 \text{ kJ/kg}]$$

$$W_{turbine} = h_1 - h_2 = 3159.3 - 2187.68 = 971.62 \text{ kJ/kg}$$

$$\therefore \quad W_{net} = 971.62 - 2.008 = \mathbf{969.61 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

Cycle efficiency, η_{cycle} :

$$Q_1 = h_1 - h_{f_4} = 3159.3 - 175.89 = 2983.41 \text{ kJ/kg}$$

$$\therefore \quad \eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{969.61}{2983.41} = \mathbf{0.325 \text{ or } 32.5\%.} \quad \textbf{(Ans.)}$$

Example 12.3 *A steam power plant has boiler and condenser pressures of 60 bar and 0.1 bar, respectively. Steam coming out of the boiler is dry and saturated. The plant operates on the Rankine cycle. Calculate thermal efficiency.*

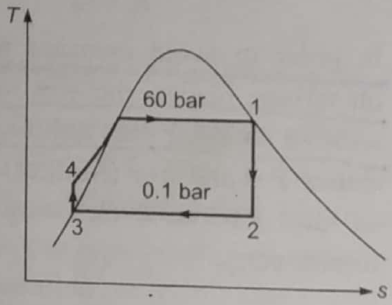

Fig. 12.6 *Rankine cycle using saturated steam*

Solution

Given Rankine cycle with dry saturated steam

$$p_1 = 60 \text{ bar} = 6000 \text{ kPa},$$
$$p_2 = 0.1 \text{ bar} = 10 \text{ kPa}$$

To find Thermal efficiency of steam power plant.

Analysis *Properties of steam at principal states*

State 1: Dry saturated steam; from Table A-13

$$\begin{aligned} p_1 &= 6000 \text{ kPa}, \\ h_1 &= 2785.10 \text{ kJ/kg} \\ s_1 &= 5.8891 \text{ kJ/kg}\cdot\text{K} \end{aligned}$$

State 2 : Wet steam;

$$\begin{aligned} p_2 &= 10 \text{ kPa} \\ h_{f_2} &= 191.81 \text{ kJ/kg} \\ h_{fg_2} &= 2392.82 \text{ kJ/kg} \\ s_{f_2} &= 0.6492 \text{ kJ/kg}\cdot\text{K} \\ s_{fg_2} &= 7.5010 \text{ kJ/kg}\cdot\text{K} \end{aligned}$$

State 3: Saturated liquid;

$$\begin{aligned} p_3 &= 0.1 \text{ bar} = 10 \text{ kPa} \\ h_3 &= h_{f_3} = 191.81 \text{ kJ/kg} \\ v_{f_3} &= 0.001010 \text{ m}^3\text{/kg} \end{aligned}$$

State 4: Compressed liquid:

$$p_4 = 6000 \text{ kPa}.$$

The state 2, after isentropic expansion can be defined by equating entropy at states 1 and 2:

$$\begin{aligned} s_1 &= s_2 = (s_f + x s_{fg})_{@\ 10 \text{ kPa}} \\ 5.8891 &= 0.6492 + x(7.5010) \end{aligned}$$

or
$$x = \frac{5.8891 - 0.6492}{7.5010} = 0.698$$

Specific enthalpy at the state 2

$$\begin{aligned} h_2 &= (h_{f_2} + x h_{fg_2})_{@\ 10 \text{ kPa}} \\ &= 191.81 + 0.698 \times 2392.82 \\ &= 1863.34 \text{ kJ/kg} \end{aligned}$$

The pump work;

$$\begin{aligned} w_p &= v_f(p_1 - p_2) = 0.001010 \times (6000 - 10) \\ &= 6.05 \text{ kJ/kg} \end{aligned}$$

Enthalpy at the state 4;

$$\begin{aligned} h_4 &= h_3 + w_p = 191.81 + 6.05 \\ &= 197.86 \text{ kJ/kg}. \end{aligned}$$

Rankine cycle efficiency, Eq. (12.16)

$$\begin{aligned} \eta_{Rankine} &= 1 - \frac{h_2 - h_3}{h_1 - h_4} = 1 - \frac{1863.34 - 191.81}{2785.10 - 197.86} \\ &= 0.353 \quad \text{or} \quad \mathbf{35.35\%} \end{aligned}$$

Example 12.4 *A steam power plant works between pressures of 40 bar and 0.05 bar. If the steam supplied is dry saturated and the cycle of operation is Rankine cycle, find*

(a) Cycle efficiency

(b) Specific steam consumption

Solution

Given Rankine cycle with dry saturated steam

$$p_1 = 40 \text{ bar} = 4000 \text{ kPa},$$
$$p_2 = 0.05 \text{ bar} = 5 \text{ kPa}$$

To find

(i) Rankine cycle efficiency, and

(ii) Specific steam consumption.

Analysis *Properties of steam at principal states (From table A-13)*

State 1: Dry saturated steam;

$$\begin{aligned} p_1 &= 4000 \text{ kPa} \\ h_1 &= 2800.36 \text{ kJ/kg} \\ s_1 &= 6.0685 \text{ kJ/kg}\cdot\text{K} \end{aligned}$$

State 2: Wet steam;

$$\begin{aligned} p_2 &= 5 \text{ kPa} \\ h_{f_2} &= 137.79 \text{ kJ/kg} \\ h_{fg_2} &= 2423.66 \text{ kJ/kg} \\ s_{f_2} &= 0.4763 \text{ kJ/kg}\cdot\text{K} \\ s_{fg_2} &= 7.9187 \text{ kJ/kg}\cdot\text{K} \end{aligned}$$

State 3: Saturated liquid;

$$\begin{aligned} p_3 &= 5 \text{ kPa}; \\ h_3 &= h_{f_3} = 137.79 \text{ kJ/kg} \\ v_{f_3} &= 0.001005 \text{ m}^3\text{/kg} \end{aligned}$$

Review Questions

1. What are four basic components of a steam power plant? Write their function in brief.
2. What do you understand by steam rate and heat rate? What are their units?
3. Why Carnot cycle is not practical for a steam power plant?
4. Explain how the quality of steam at turbine exit gets restricted?
5. Draw the schematic for an ideal Rankine cycle. Draw p–v, T–s and h–s diagrams for this cycle.
6. What are methods which can lead to increase in thermal efficiency of Rankine cycle?
7. What are the irreversibilities in a steam power plant, which make its thermal efficiency less than that of Rankine cycle?

8. What is reheating? What are the advantage of reheat Rankine cycle?

9. What is the effect of reheating of steam on (a) specific power output, (b) cycle efficiency, and (c) steam rate?

10. What is regeneration? Draw schematic and T–s diagram for an ideal regenerative cycle.

11. Why ideal regeneration is not possible? Explain in brief.

12. What is the effect of regeneration on (a) specific power output, (b) cycle efficiency, and (c) steam rate?

13. How is the regeneration of steam done in Carnotisation of Rankine cycle?

14. Explain the working and analysis of the regenerative Rankine cycle with one feed-water heater.

Problems

1. In a steam power cycle, the steam supply is at 15 bar, dry and saturated. The condenser pressure is 0.4 bar. Calculate the Carnot and Rankine efficiencies of the cycle. Neglect pump work.
[25.9%, 23.54%]

2. A steam power plant operates between a boiler pressure of 42 bar and a condenser pressure of 0.035 bar. The steam enters the turbine just dry and saturated. Calculate for these limits the cycle efficiency, work ratio, and specific steam consumption for
   (a) Carnot cycle
   (b) Ideal Rankine cycle
   (c) Rankine cycle when expansion process has an isentropic efficiency of 80%
[(a) 42.2%,0.739, 4.91 kg/kWh, (b) 36.8%, 0.996, 3.64 kg/kWh, (c) 29.4%, 0.995, 4.56 kg/kWh]

3. A simple Rankine cycle works between pressures of 28 bar and 0.06 bar, the initial condition of steam being dry saturated. Calculate the cyclic efficiency, work ratio and specific steam consumption rate. *[33.57%, 0.997, 4.049 kg/kWh]*

4. An ideal Rankine cycle uses superheated steam at 50 bar and 500°C. The condenser pressure is 0.05 bar. Calculate cycle efficiency and specific steam consumption. *[39.6%, 2.75 kg/kWh]*

5. A steam turbine receives steam at 100 bar and 600°C. It is exhausted steam at 2 bar. For the ideal Rankine cycle, calculate net work, specific steam consumption, cycle efficiency and mean effective pressure.
[995.5 kJ/kg, 3.62 kg/kWh, 32.03%, 11.71 bar]

6. A steam power plant operates on ideal Rankine cycle, receives steam at 20 bar and 300°C at a rate of 3 kg/s and it is exhausted at 0.1 bar. Calculate the followings
   (a) Net power output
   (b) Steam rate
   (c) Heat rejected in the condenser in kW
   (d) Rankine cycle efficiency
   (e) Actual thermal efficiency of the plant, if the boiler efficiency is 90%
[(a) 2636.4 kW, (b) 4.1 kg/kWh, (c) 5857.2 kW, (d) 31.04%, (e) 27.94%]

7. Steam at 20 bar, 360°C is expanded in a turbine to 0.08 bar. It then enters a condenser where it is condensed to saturated liquid water. The feed pump supplies saturated water back to the boiler.
   (a) Calculate the net work per kg of steam and cycle efficiency for ideal Rankine cycle.
   (b) If the turbine and pump have each 80% isentropic efficiency, calculate percentage reduction in net work and cycle efficiency.
[(a) 2983.41 kJ/kg, 32.55, (b) 20.1%, 20.1%]

8. A steam power plant operating on Rankine cycle is supplied with steam at 10 bar and 200°C. The condenser vacuum is 600 mm of mercury. Barometer pressure is 750 mm of Hg. If the efficiency of the plant relative to Rankine cycle is 65%, calculate (a) specific steam consumption, and (b) work ratio.
[(a) 8.888 kg/kWh, (b) 0.998]

9. A turbine used in a simple Rankine cycle receives steam at 15 bar with 5% moisture. The steam enters the condenser at a temperature of 29°C. Calculate Rankine cycle efficiency.
[31.77%]